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σ-ω model and compressible bag model*

S. Kagiyama[1], A. Nakamura[2], T. Otofuji[3]

[1] Department of Physics, Faculty of Science, Kagoshima University, Kagoshima 890, Japan (e-mail: kagiyama@sci.kagoshima-u.ac.jp)
[2] Department of Physics, Faculty of Science, Kagoshima University, Kagoshima 890, Japan (e-mail: nakamura@sci.kagoshima-u.ac.jp)
[3] Department of Physics, College of Education, Akita University, Akita 010, Japan (e-mail: otofuji@phys.akita-u.ac.jp)

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Abstract. Compressible bag model is formulated on the basis of lagrangian field theory. A specific application is done in conjunction with σ-ω model. The results are similar to Chin-Walecka model and almost reproduce our previous results. The effective nucleon mass does not become so small owing to the compressibility, in contrast to Chin-Walecka model.

1 Introduction

Recently, physics of quark matter and hadronic matter at finite density and/or finite temperature is of interest and is planned to investigate experimentally in higher energy density region by RHIC and LHC. On the theoretical side, equation of state for hadronic matter at finite density and finite temperature is necessary to study such physics as deconfinement transition, ultrarelativitic heavy-ion collisions, neutron star and early universe. For high density hadronic matter, the effect of relativity and the effect of size of hadron become important. As for relativistic effect, the fermi momentum of nucleon is already 250 MeV at normal density, comparable to the mass of nucleon in nuclear/neutron matter. Chin and Walecka [1–3] formulated fully relativistically σ-ω model, hereafter we refer it as CW-model, and succeeded to explain gross feature of nuclear physics at normal density. As for size effect, nucleon charge radius is about 1 fm and nuclear matter cannot exist in the high density region above the baryon number density of the inverse of the bag if nucleon has fixed size and cannot overlap.

In the previous paper [4], we proposed a compressible bag model that presents a consistent equation of state for hadrons applicable at any density. In the compressible bag model, a bag in many body system feels the microscopic thermal pressure from the other bags and is compressed to minimize the total free energy of the system. Owing to the compressibility, hadronic matter can exist at higher density than inverse volume. We have applied the compressible bag model to several problems to obtain satisfactory results [5–7]. The key object to formulate compressible bag model is the bag volume dependent free energy $\tilde{F}$. Once the form of $\tilde{F}$ is given one can develop thermodynamics of many body system of bags. In the previous paper, we adopted a phenomenological form of relativistic fermi gas with particle exchange type interactions and applied to nuclear physics at normal density and astrophysics at high density.

The purpose of the present paper is to derive $\tilde{F}$ on the basis of the lagrangian field theory and to obtain the equation of state using the derived $\tilde{F}$. The specific application is done in conjunction with σ-ω model. The results almost reproduce those of the previous paper. The results are also similar to CW-model. However, the stiffness of equation of state is different. In the compressible bag model, the repulsive force due to the volume exclusion effect can sustain the attraction by σ exchange interaction, so that it can take the place of ω exchange repulsive force. Then the asymptotic behavior of the energy per nucleon in the compressible bag model is proportional to $\rho^{1/3}$ (ρ is the density) while that is proportional in CW-model to ρ. A remarkable difference lies in the behavior of the effective mass of nucleon [8,9]. In CW-model, the nucleon mass decreases continuously with density and shows rather small effective mass even in the normal density region. In the compressible bag model, the nucleon mass becomes larger as nucleon bag shrinks when the density becomes higher. Then the effective mass of nucleon turn to increase and the model reproduces a usual value for the effective mass of nucleon.

In Sect. 2, the formulation is presented and the results of its application is given in Sect. 3. The final section is devoted to discussion.

2 The formulation

Let us start with general formalism. Suppose field operator be ϕ_i and its mass parameter be M_i for the i-th kind

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particle and coupling constant be G_I at the I-th kind of interaction. We assume all the interactions to be local. The lagrangian density is given by

$$\mathcal{L} = \mathcal{L}(\phi_i, \partial_\mu \phi_i; M_i, G_I). \tag{1}$$

In order to get free energy, it is necessary to introduce number operator $\hat{N}_i$ and to add source term to lagrangian

$$\int d^3\mathbf{x}\, \mathcal{L}(\phi_i, \partial_\mu \phi_i; M_i, G_I) + \sum_i \mu_i \hat{N}_i, \tag{2}$$

where μ_i is the chemical potential to $\hat{N}_i$. The number operators are defined as sums of products of a creation operator and an annihilation operator, taking interaction picture. Standard finite temperature field theory [10] tells us a thermodynamical potential;

$$e^{\beta pV} = \int [D\phi_i] \exp\left[\int_0^\beta d\tau \left\{\int d^3\mathbf{x}\, \mathcal{L}(\phi_i, \partial_\mu \phi_i; M_i, G_I) + \sum_i \mu_i \hat{N}_i \right\}\right], \tag{3}$$

where p is the pressure, V the volume and β the inverse temperature. The pressure is given by

$$p = p(\mu_i, T, M_i, G_I). \tag{4}$$

The free energy is given by the Legendre transformation,

$$F(N_i, V, T, M_i, G_I) = \sum_i \mu_i N_i - pV, \tag{5}$$

with

$$N_i = V \frac{\partial p}{\partial \mu_i}. \tag{6}$$

In order to translate the free energy (5) to the one for the compressible bag model, we consider the free energy that is a sum of a free part and an interaction part,

$$F = \textstyle\sum_i F_f(N_i, V, T, M_i) + F_{\text{int}}(N_i, V, T, M_i, G_I). \tag{7}$$

In the compressible bag model, the bag feels microscopic thermal pressure by other bags. This effect is taken into account by the replacement

$$V \to V' = V - b \textstyle\sum_i N_i v_i, \tag{8}$$

in the free part of F, where b is volume exclusion efficiency parameter and v_i is the bag volume. A few comments are in order. The first comment is about the reason why we have added the source terms for number operators of all fields in (2). The reason is that it is necessary for the replacement (8) to keep all number variables in the free energy as independent variables. At the end of calculation, the chemical potential corresponding to non-conserving number should be set to zero. The second comment is that all the hadron are treated on equal footing. That is, not only nucleon but also all other hadrons including mesons are treated on the basis of bag model so that all the hadrons contribute to volume exclusion (8). The last comment is that the replacement (8) should not be done in the interaction part. The reason is as follows. Since we have assumed all the interactions in the lagrangian (1) to be local, interactions take place at a spacetime point in the level of elementary hadron interactions coded by the lagrangian. When the elementary hadrons are replaced with bags, it arises a picture that bags should meet and overlap in order to interact one another. If the bag model will be derived from QCD, the bag interactions will take place by exchanging gluons inside the overlapped bag. Therefore the volume exclusion does not take place in interactions originating from the lagrangian (1). Even if the free energy cannot be separated as a sum of a free part and an interaction part, the interaction terms may be distinguished from other terms by the appearance of coupling constants. Then the free energy for the compressible bag model is obtained by the replacement (8) in the terms other than interaction terms.

The hadron mass should be replaced with that of a bag model,

$$M_i = M_i(v_i). \tag{9}$$

The coupling constant should be replaced with a vertex function. Its most general form is given by

$$G_I = G_I(v_i, N_i/V, T). \tag{10}$$

In the below, we will adopt MIT form for $M_i(v_i)$ for the purpose of specific application. As for the vertex function, we will assume it to be a constant for simplicity. The free energy with replacements of (8), (9) and (10) is the starting point of the compressible bag model. To remind the fact that the replacement (8) should not be done in the terms containing coupling constants, we attach a prime to the free energy function (5),

$$\tilde{F} = F'(N_i, V', T, M_i(v_i), G_I(v_i, N_i/V, T)). \tag{11}$$

Minimization conditions with respect to bag volume and the conditions that the chemical potentials μ_n corresponding to non-conserving number N_n should vanish,

$$\frac{\partial \tilde{F}}{\partial v_i} = 0, \qquad \mu_n = \frac{\partial \tilde{F}}{\partial N_n} = 0, \tag{12}$$

determine v_i and N_n as functions of T, V and conserving numbers such as baryon number. The substitution of the solutions into the free energy function (11) yields the final free energy. Then we can develop thermodynamics of the system.

Now we apply the compressible bag model in conjunction with σ-ω model. The lagrangian for σ-ω model is defined by

$$L = \int d^3\mathbf{x} \left\{ \bar{\psi}[\gamma^\mu(i\partial_\mu - G_\omega \omega_\mu) + G_\sigma \sigma - M]\psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}M_\sigma^2 \sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_\omega^2 \omega_\mu \omega^\mu \right\}, \tag{13}$$

where $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ is the field strength for ω field. To get free energy, we add source terms

$$L \to L + \mu_T \hat{N}_T + \mu_B \hat{N}_B + \mu_\sigma \hat{N}_\sigma + \mu_\omega \hat{N}_\omega, \quad (14)$$

where $\hat{N}_T$, $\hat{N}_B$ $\hat{N}_\sigma$ and $\hat{N}_\omega$ represent operators for total nucleon number, baryon number, omega number and sigma number, respectively. Taking interaction picture, we define the number operators as sums of products of a creation operator and an annihilation operator. Explicit forms of definitions are as follows:

$$\hat{N}_T = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{M}{k^0} \sum_s \left[b^\dagger(\mathbf{k},s) b(\mathbf{k},s) + d^\dagger(\mathbf{k},s) d(\mathbf{k},s) \right] \quad (15)$$

$$\hat{N}_B = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{M}{k^0} \sum_s \left[b^\dagger(\mathbf{k},s) b(\mathbf{k},s) - d^\dagger(\mathbf{k},s) d(\mathbf{k},s) \right] \quad (16)$$

$$\begin{aligned} b(\mathbf{k},s) &= \int d^3\mathbf{x}\, u^\dagger(\mathbf{k},s) e^{ikx} \psi(x), \\ d(\mathbf{k},s) &= \int d^3\mathbf{x}\, \psi^\dagger(x) v(\mathbf{k},s) e^{ikx}, \\ b^\dagger(\mathbf{k},s) &= \int d^3\mathbf{x}\, \psi^\dagger(x) u(\mathbf{k},s) e^{-ikx}, \\ d^\dagger(\mathbf{k},s) &= \int d^3\mathbf{x}\, v^\dagger(\mathbf{k},s) e^{-ikx} \psi(x) \end{aligned} \quad (17)$$

$$\hat{N}_\sigma = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0} a^\dagger(\mathbf{k}) a(\mathbf{k}) \quad (18)$$

$$\begin{aligned} a(\mathbf{k}) &= i \int d^3\mathbf{x}\, e^{ikx} \overleftrightarrow{\partial_0} \sigma(x), \\ a^\dagger(\mathbf{k}) &= -i \int d^3\mathbf{x}\, e^{-ikx} \overleftrightarrow{\partial_0} \sigma(x) \end{aligned} \quad (19)$$

$$\hat{N}_\omega = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0} \sum_s c^\dagger(\mathbf{k},s) c(\mathbf{k},s) \quad (20)$$

$$\begin{aligned} c(\mathbf{k},s) &= -i \int d^3\mathbf{x}\, \epsilon^\mu(\mathbf{k},s) e^{ikx} \overleftrightarrow{\partial_0} \omega_\mu(x), \\ c^\dagger(\mathbf{k},s) &= i \int d^3\mathbf{x}\, \epsilon^\mu(\mathbf{k},s) e^{-ikx} \overleftrightarrow{\partial_0} \omega_\mu(x) \end{aligned} \quad (21)$$

$$\begin{aligned} \epsilon_\mu(\mathbf{k},s) \epsilon^\mu(\mathbf{k},s') &= -\delta_{ss'}, \\ \sum_s \epsilon^\mu(\mathbf{k},s) \epsilon^\nu(\mathbf{k},s) &= -g^{\mu\nu} + \frac{k^\mu k^\nu}{M_\omega^2}. \end{aligned} \quad (22)$$

In the above $k^0 = \sqrt{\mathbf{k}^2 + m^2}$ and $b^\dagger$ ($d^\dagger$) and b (d) are creation operator and annihilation operator for nucleon (antinucleon) and $a^\dagger$ and a are creation operator and annihilation operator for σ-particle and $c^\dagger$ and c are creation operator and annihilation operator for ω-partile. Their normalization conventions and other notations are the same as those in the textbook of Itzykson and Zuber [11]. Since $\hat{N}_T$, $\hat{N}_\sigma$ and $\hat{N}_\omega$ are not conserved quantity (in Heisenberg picture), we set corresponding chemical potentials to zero at the end of calculation.

Under mean field approximation for σ and ω, translational invariance and rotational invariance require that

$$\begin{aligned} &\langle \sigma(x) \rangle = \sigma = \text{const.}, \qquad \langle \omega_0(x) \rangle = \omega_0 = \text{const.}, \\ &\langle \omega_i(x) \rangle = 0. \end{aligned} \quad (23)$$

Using (18)∼(22) and (23), vacuum expectation values of $\hat{N}_\sigma$ and $\hat{N}_\omega$ are calculated as follows;

$$\begin{aligned} \langle \hat{N}_\sigma \rangle &= \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0} \int d^3\mathbf{x} \int d^3\mathbf{x}' e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} k_0^2 \sigma^2 \\ &= \frac{1}{2} M_\sigma \sigma^2 V, \\ \langle \hat{N}_\omega \rangle &= \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0} \int d^3\mathbf{x} \int d^3\mathbf{x}' \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_\omega^2} \right) \\ &\quad \times e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} k_0^2 \omega^\mu \omega^\nu \\ &= -\frac{1}{2} M_\omega g_{ij} \omega^i \omega^j V = 0. \end{aligned} \quad (24)$$

It should be noted that time component of ω-field does not contribute to the vacuum expectation value of its number operator. Physically, (24) means that σ is condensed in the zero mode (bose condensation) while ω is not. Owing to (23) and (24), the fermion part of (14) reduces to the free lagrangian with the following replacement

$$\mu_B \to \mu_B - G_\omega \omega_0, \qquad M \to M^* = M - G_\sigma \sigma, \quad (25)$$

and the statistical sum is trivially performed to give

$$\begin{aligned} p = -\Omega/V &= \frac{1}{\beta V} \ln \mathrm{Tr}\, e^{\beta L} \\ &= p_f(\mu'_N, T, M^*) + p_f(\mu'_{\bar{N}}, T, M^*) \\ &\quad - \frac{1}{2} M_\sigma^2 \sigma^2 + \frac{1}{2} M_\sigma \sigma^2 \mu_\sigma + \frac{1}{2} M_\omega^2 \omega_0^2, \end{aligned} \quad (26)$$

with

$$\begin{aligned} \mu'_N &= \mu_T + \mu_B - G_\omega \omega_0, \\ \mu'_{\bar{N}} &= \mu_T - \mu_B + G_\omega \omega_0, \end{aligned} \quad (27)$$

where p_f denotes the pressure for free fermi gas. We write down here the function form of p_f, as well as its Legendre transform to free energy F_f for later convenience,

$$p_f(\mu, T, m) = gT \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln \left[1 + e^{-(E-\mu)/T} \right], \quad (28)$$

$$\begin{aligned} &E = \sqrt{\mathbf{k}^2 + m^2}, \\ &N = V p_{f,\mu}(\mu, T, m) = gV \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{e^{(E-\mu)/T} + 1}, \end{aligned} \quad (29)$$

$$F_f(N, V, T, m) = \mu N - p_f(\mu, T, m) V, \quad (30)$$

where g is a degeneracy factor. Mean field requirements $\partial p/\partial\sigma = \partial p/\partial\omega_0 = 0$ are written as

$$\partial p/\partial\sigma = -(M_\sigma^2 - M_\sigma\mu_\sigma)\sigma + G_\sigma\rho_S = 0, \quad (31)$$
$$\partial p/\partial\omega_0 = M_\omega^2\omega_0 - G_\omega\rho_B = 0, \quad (32)$$

with

$$\rho_S = g\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{M^*}{E} \times\left[\frac{1}{e^{(E-\mu'_N)/T}+1}+\frac{1}{e^{(E-\mu'_{\bar{N}})/T}+1}\right], \quad (33)$$
$$\rho_B = \frac{\partial p}{\partial\mu_B} = p_{f,\mu}(\mu'_N,T,M^*) - p_{f,\mu}(\mu'_{\bar{N}},T,M^*), \quad (34)$$

where ρ_S is nucleon scalar density while ρ_B baryon number density. Performing a Legendre transformation of p with the aid of (31) and (32),

$$N_T = V\frac{\partial p}{\partial\mu_T} = V\left[p_{f,\mu}(\mu'_N,T,M^*) + p_{f,\mu}(\mu'_{\bar{N}},T,M^*)\right],$$
$$N_B = V\frac{\partial p}{\partial\mu_B} = V\left[p_{f,\mu}(\mu'_N,T,M^*) - p_{f,\mu}(\mu'_{\bar{N}},T,M^*)\right],$$
$$N_\sigma = V\frac{\partial p}{\partial\mu_\sigma} = V\frac{1}{2}M_\sigma\sigma^2 = V\frac{G_\sigma^2 M_\sigma\rho_S^2}{2(M_\sigma^2 - M_\sigma\mu_\sigma)}, \quad (35)$$

we obtain free energy

$$\begin{aligned} F &= \mu_T N_T + \mu_B N_B + \mu_\sigma N_\sigma - pV \\ &= \mu'_N N_N + \mu'_{\bar{N}} N_{\bar{N}} + G_\omega\omega_0 N_B + \mu_\sigma N_\sigma - pV \\ &= F_f(N_N,V,T,M^*) + F_f(N_{\bar{N}},V,T,M^*) \\ &\quad + M_\sigma N_\sigma + \frac{G_\omega^2 N_B^2}{2M_\omega^2 V}, \\ &\equiv F(N_N,N_{\bar{N}},N_\sigma,V,T,M,M_\sigma,G_\sigma,G_\omega), \end{aligned} \quad (36)$$

$$M^* = M - G_\sigma\left(\frac{2N_\sigma}{M_\sigma V}\right)^{1/2},$$
$$N_N \equiv \frac{1}{2}(N_T + N_B), \qquad N_{\bar{N}} \equiv \frac{1}{2}(N_T - N_B). \quad (37)$$

In the above, we have introduced the notation $N_N(N_{\bar{N}})$ for nucleon (antinucleon) number.

Following the general consideration mentioned above, one may obtain v-dependent free energy $\tilde{F}$ from (36) as

$$\tilde{F} = F'(N_N,N_{\bar{N}},N_\sigma,V',T,M(v),M_\sigma(v_\sigma),G_\sigma,G_\omega), \quad (38)$$
$$V' = V - b(N_N v + N_{\bar{N}} v + N_\sigma v_\sigma). \quad (39)$$

The masses are given by MIT bag model as

$$M(v) = A_N v^{-1/3} + Bv,\ M_\sigma(v_\sigma) = A_\sigma v_\sigma^{-1/3} + Bv_\sigma. \quad (40)$$

It should be noted that the volumes of ω particles are not subtracted in (39) as a consequence of the second equation in (24). Since disappearance of v_ω in V' means that ω-bags do not feel external microscopic pressure by other hadrons, M_ω is not replaced with that of bag model in (40). (See also the comment after (46).) The vertex functions are assumed to be constants for simplicity,

$$G_\sigma = g_\sigma, \qquad G_\omega = g_\omega. \quad (41)$$

In the below, we will confine ourselves to the discussion for the case of $T = 0$. Replacing N_B with N, the form of internal energy function $\tilde{E}$ is given by

$$\tilde{E} = E_f(N,V',M^*) + M_\sigma(v_\sigma)N_\sigma + \frac{g_\omega^2 N^2}{2M_\omega^2 V}. \quad (42)$$
$$V' = V - b(Nv + N_\sigma v_\sigma), \quad (43)$$
$$M^* = M(v) - g_\sigma\left(\frac{2N_\sigma}{M_\sigma(v_\sigma)V}\right)^{1/2}, \quad (44)$$

with

$$\begin{aligned} E_f(N,V,m) &= F_f(N,V,T=0,m) \\ &= gV\int_{k\le k_F}\frac{d^3\mathbf{k}}{(2\pi)^3}(\mathbf{k}^2+m^2)^{1/2}, \\ k_F &= (6\pi^2 N/gV)^{1/3}. \end{aligned} \quad (45)$$

Now we may impose the compressible bag model requirements on the internal energy (42) and the condition that the chemical potential μ_σ corresponding to nonconserving σ-number N_σ should vanish

$$\frac{\partial\tilde{E}}{\partial v} = \frac{\partial\tilde{E}}{\partial v_\sigma} = 0, \qquad \mu_\sigma = \frac{\partial\tilde{E}}{\partial N_\sigma} = 0. \quad (46)$$

In the above, we have not imposed a minimization condition with respect to v_ω since ω particles are absent. If we imposed it with mass function for ω particle replaced with that of bag model, we would get result that the volume of ω particle is the same as that in the vacuum. Therefore we have set the mass of ω to the value in the vacuum from the outset. We can obtain the equation of states on the basis of (42)∼(46).

The dependence of the resulting equation of state on g_σ and b comes in the combination of $g_\sigma/M_{\sigma 0}$ and b/B. This is seen as follows. Changing variables v, v_σ and N_σ to

$$\tilde{v}_i = \frac{v_i}{v_{i0}}, \qquad \tilde{N}_\sigma = M_{\sigma 0}N_\sigma, \quad (47)$$

and using the relation,

$$v_{i0} = \frac{M_{i0}}{4B}, \quad (48)$$

where subscript 0 denotes values in the vacuum, $\tilde{E}$ is rewritten as

$$\begin{aligned} \tilde{E} &= E_f(N,V',M^*) + (M_\sigma(v_\sigma)/M_{\sigma 0})\tilde{N}_\sigma \\ &\quad + \frac{g_\omega^2 N^2}{2M_\omega^2 V}, \end{aligned} \quad (49)$$

$$\begin{aligned} V' &= V - b(Nv_0\tilde{v} + (\tilde{N}_\sigma/M_{\sigma 0})v_{\sigma 0}\tilde{v}_\sigma) \\ &= V - (b/4B)(NM_0\tilde{v} + \tilde{N}_\sigma\tilde{v}_\sigma), \end{aligned} \quad (50)$$

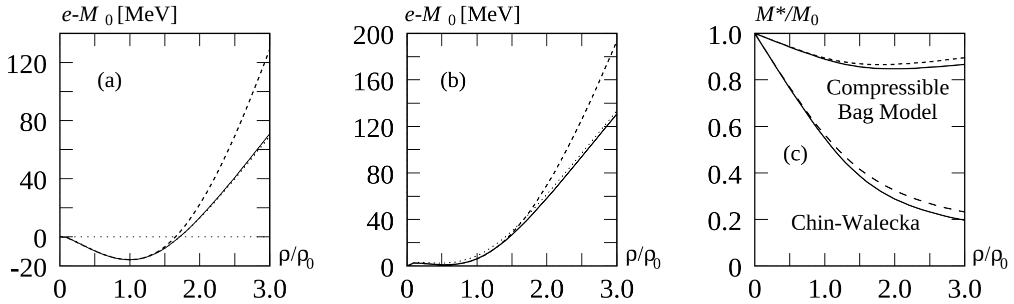

Fig. 1. Density dependence of the energy for symmetric nuclear matter and neutron mater are shown in **a** and **b**, respectively. Solid, dotted and dashed lines depicts the results for present, previous and Chin-Walecka models, respectively. In **c**, density dependence of the effective mass M^*/M_0 is shown. Solid lines shows for the symmetric nuclear matter, dashed for the neutron matter. The upper group corresponds to present result, and the lower to the Chin-Walecka model

$$
\begin{aligned}
M^* &= \frac{M(v)}{M_0} M_0 - g_\sigma \left(\frac{2\tilde{N}_\sigma / M_{\sigma 0}}{(M_\sigma(v_\sigma)/M_{\sigma 0}) M_{\sigma 0} V} \right)^{1/2} \\
&= \frac{M(v)}{M_0} M_0 - \frac{g_\sigma}{M_{\sigma 0}} \left(\frac{2\tilde{N}_\sigma}{(M_\sigma(v_\sigma)/M_{\sigma 0}) V} \right)^{1/2} \qquad (51)
\end{aligned}
$$

and $M_i(v_i)/M_{i0}$ is written as

$$
\frac{M_i(v_i)}{M_{i0}} = \frac{1}{4}(3\tilde{v}_i^{-1/3} + \tilde{v}_i). \qquad (52)
$$

Thus it is clearly seen from (49)∼(52) that b and g_σ enter in $\tilde{E}$ with the combination of b/B and $g_\sigma/M_{\sigma 0}$, respectively.

3 Applications and their results

Now let us apply the equations of states to nuclear physics at normal density, deconfinement transition and neutron stars. The total energy of the nucleus of atomic number Z and mass number A is given by

$$
\begin{aligned}
\tilde{E} &= E_f(Z, V', M^*) + E_f(A - Z, V', M^*) \\
&\quad + M_\sigma N_\sigma + \frac{g_\omega^2 A^2}{2 M_\omega^2 V} , \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
M^* &= M - g_\sigma \left(\frac{2N_\sigma}{M_\sigma V} \right)^{1/2}, \\
V' &= V - b(Av + N_\sigma v_\sigma) \qquad (54)
\end{aligned}
$$

with

$$
\begin{aligned}
E_f(N, V, m) &= gV \int_{k \le k_F} \frac{d^3\mathbf{k}}{(2\pi)^3} (\mathbf{k}^2 + m^2)^{1/2} \\
&= \frac{gV}{24\pi^2} \Big[3k_F^3 (k_F^2 + m^2)^{1/2} \\
&\quad + \frac{3}{2} m^2 k_F (k_F^2 + m^2)^{1/2} \\
&\quad - \frac{3}{2} m^4 \ln \frac{k_F + (k_F^2 + m^2)^{1/2}}{m} \Big], \\
k_F &= (6\pi^2 N / gV)^{1/3}, \quad g \equiv 2. \qquad (55)
\end{aligned}
$$

The parameters to be determined are those related to bag model, and those related to nuclear matter properties. The parameters related to bag model are A_N, A_σ and B. Here A_σ need not to be determined since $M_{\sigma 0}$ appear in the combination of $g_\sigma/M_{\sigma 0}$ as mentioned in the bottom of the last section. The values of A_N and B are determined by using experimental values of nucleon mass M_0 and the proton charge radius R_0

$$
M_0 = 0.94\,\mathrm{GeV}, \qquad R_0 = 0.82\,\mathrm{fm}, \qquad (56)
$$

as

$$
A_N = 4.7, \qquad B = (0.167\,\mathrm{GeV})^4. \qquad (57)
$$

Two of the remaining parameters, b, $g_\sigma/M_{\sigma 0}$, g_ω/M_ω are determined by the normal nuclear density ρ_0 and the binding energy u_V,

$$
\rho_0 = 0.17\,\mathrm{fm}^{-3}, \qquad u_V = 15.7\,\mathrm{MeV}. \qquad (58)
$$

The pressure are calculated using $p = -E_V$

$$
\begin{aligned}
p &= p_f\left(\frac{Z}{A}\rho', M^*\right) + p_f\left(\frac{A-Z}{A}\rho', M^*\right) \\
&\quad - \frac{1}{2}\rho_S g_\sigma \left(\frac{2\rho_\sigma}{M_\sigma}\right)^{1/2} + \frac{g_\omega^2}{2M_\omega^2}\rho^2, \\
\rho &= A/V, \qquad \rho_\sigma = N_\sigma / V. \qquad (59)
\end{aligned}
$$

The attraction by σ and the repulsion by ω cancels and balances the nuclear matter in CW-model. In the compressible bag model, the repulsion is coming from not only ω but also pressure caused by volume exclusion. And, as by only volume exclusion the normal nuclear matter properties can be reproduced, we omit here the contribution of the ω meson by setting $g_\omega/M_\omega = 0$ for simplicity.

The results are shown in Fig. 1. In Fig. 1a and b, e denotes an energy per nucleon. For the density dependence of the energy, the present and previous result are almost same. In CW-model, energy is increased rapidly when the density increases compared with our results. The density dependence of the effective mass is shown in Fig. 1c. In CW-model, M^* decreases continuously with density. While, M^* shows a rather large value and a minimum in the compressible bag model. The decrease of M^*

Table 1. Results for nuclear physics at normal density, deconfinement transition and neutron stars. Boldface digits are used as inputs

	previous	present	CW
b	1.53	1.28	–
$g_\sigma/M_{\sigma 0}$[GeV^{-1}]	–	9.29	17.0
g_ω^2/M_ω^2 [GeV^{-2}]	–	**0**	222
other parameters	$a_S = 37.4$	–	–
	$a_V = 8.8$		
ρ_0 [fm^{-3}]	**0.17**	**0.17**	**0.17**
u_V [MeV]	**15.7**	**15.7**	**15.7**
u_τ [MeV]	**23.6**	20.9	20.7
K [MeV]	553	538	544
nuclear matter ρ_c^H/ρ_0	10.6	9.0	2.5
ρ_c^Q/ρ_0	11.1	9.5	4.0
neutron matter ρ_c^H/ρ_0	3.2	3.3	2.1
ρ_c^Q/ρ_0	4.0	4.1	3.5
Radius[km] of 1.44 $M_\odot$	12.9	12.8	12.9
Maximum mass /$M_\odot$	2.1	2.1	2.2
Radius[km] of quark core	1.4	1.2	none

comes from the attraction by the σ field in both cases. However, as the nucleon mass is determined by the pressure balance in the compressible bag model, M^* increases in high density where pressure is high.

The other results are summarized in the Table 1, where u_τ is the symmetric energy and K is the compressibility defined by $K = 9dp/d\rho$ whose experimental value is 210 ± 10 MeV [12]. The mass of neutron star is calculated using TOV equation [13]. As seen from the table, the results are very similar to the previous results. Except for critical densities for the deconfinement transition and the quark core radius of the maximum mass neutron stars, three columns are similar to one another. The differences between the former two and the CW-model originate from the existence of ω exchange potential which has stronger asymptotic behavior than the volume exclusion effect.

4 Discussions

We have shown general formulation of relativistic many body theory in which particles have finite volume, and have applied the formulation to σ-ω model with $T = 0$. Both CW-model and compressible bag model give very similar results for nuclear matter property and neutron star properties. However, the value of M^* and asymptotic density dependence of energy show large difference. These differences are reflection of the difference of the repulsion in the models. In compressible bag model the repulsive contribution to the energy are ω exchange and the volume exclusion effect. While in CW-model, only ω exchange plays a role of repulsion.

We discuss here treatments of the meson field in the compressible bag model. The first one is for the σ meson,

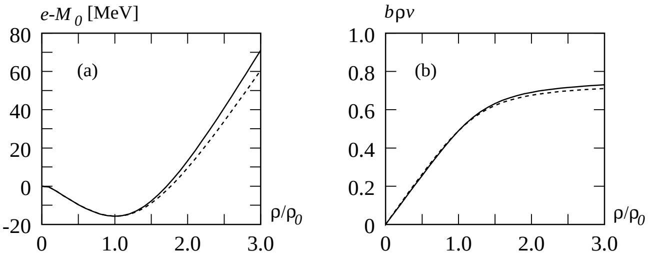

Fig. 2. Density dependence of the nucleon volume, and the ratio of excluded volume are shown in **a** and **b**, respectively. The solid line shows the result in which both of nucleon and σ exclude volume, dashed result in which only the nucleon exclude volume

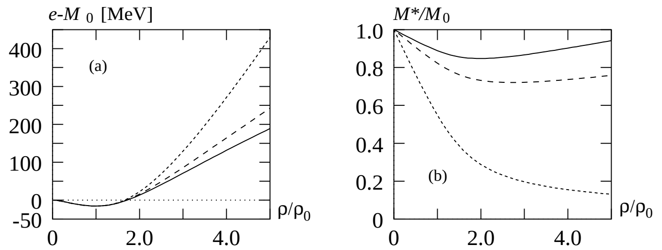

Fig. 3. Density dependence of energy and effective mass are shown in **a** and **b**, respectively. Solid, dotted and dashed lines depict the results of present model without ω coupling, CW-model and present model with ω coupling $g_\omega^2/M_\omega^2 = 50$ GeV^{-2}, respectively

especially volume exclusion of the σ meson. σ field is a background field in the mean field approximation. And as commented, the free energy depends on $M_{\sigma 0}$ only in the combination of $g_\sigma/M_{\sigma 0}$. Then it arises a question that the σ meson is real meson. To see this, nuclear matter properties are calculated with omitting the σ volume exclusion in (43), and are shown in Fig. 2. By omitting σ volume exclusion, the value of b changes and takes a little bit larger value in order to reproduce the nuclear matter property at ρ_0. Net contributions are absorbed into the value of b, and the difference between the results with σ volume exclusion and those without exclusion is small and differs about 10% at most. Therefore, from this result, it is not possible to choose the right treatment of σ meson.

The second is for the treatment of ω meson. We have omitted the ω field to show that the volume exclusion can sustain the attraction without ω exchange repulsion. It has remarked that the density dependence of the energy is different between CW-model and the compressible bag model. The difference between models is coming from whether the model includes ω field or not. In CW-model ω exchange contribution increase the energy in high density. And the energy in high density is dominated by ω exchange contribution. Therefore asymptotic density dependence of energy is proportional to ρ. While, in the compressible bag model M^* increases by the pressure in high density. As we have omit ω field M^* is determined by the pressure balance. Then M^* shows dependence of $\rho^{1/3}$, and the energy shows asymptotic behavior of $\rho^{1/3}$ in high density.

We have simply omitted the ω field as the parameters relating to ω meson are not able to be determined in the present framework. To see the effect of the ω field contribution and density dependence of energy, we calculate nuclear matter properties by taking g_ω^2/M_ω^2 to be 50 GeV^{-2}. Resulting energy and the M^* in wider density range are shown in Fig. 3. The compressible bag model gives minimum of M^* even when ω meson couples, and the value shows a smaller one. For the energy it is clear that the larger the coupling, the results approaches to those of CW-model. By only studying a nuclear matter properties, there is no clear differences except the effective mass between CW-model and compressible bag model. The critical point of deconfinement transition and the structure of the neutron star may be clues as well as nuclear force approach.

In the application to general formulation to σ-ω model, vertex functions are assumed to be constant as in (41). This assumption may be justified under the mean field approximation since the coupling between nucleon and meson takes only zero momentum transfer under the approximation so that the model is free from structure of vertex functions. This, in turn, means that the model does not determine $M_{\sigma 0}$. However, beyond mean field approximation in the case of $T \neq 0$, nucleons are directly coupled with real meson which is excited thermodynamically. In that case, $M_{\sigma 0}$ or structure of vertex functions is determined by the equation of states.

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